Computing the K_L - K_S mass difference in Lattice QCD

Jianglei Yu

RBC and UKQCD Collaborations

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Outline

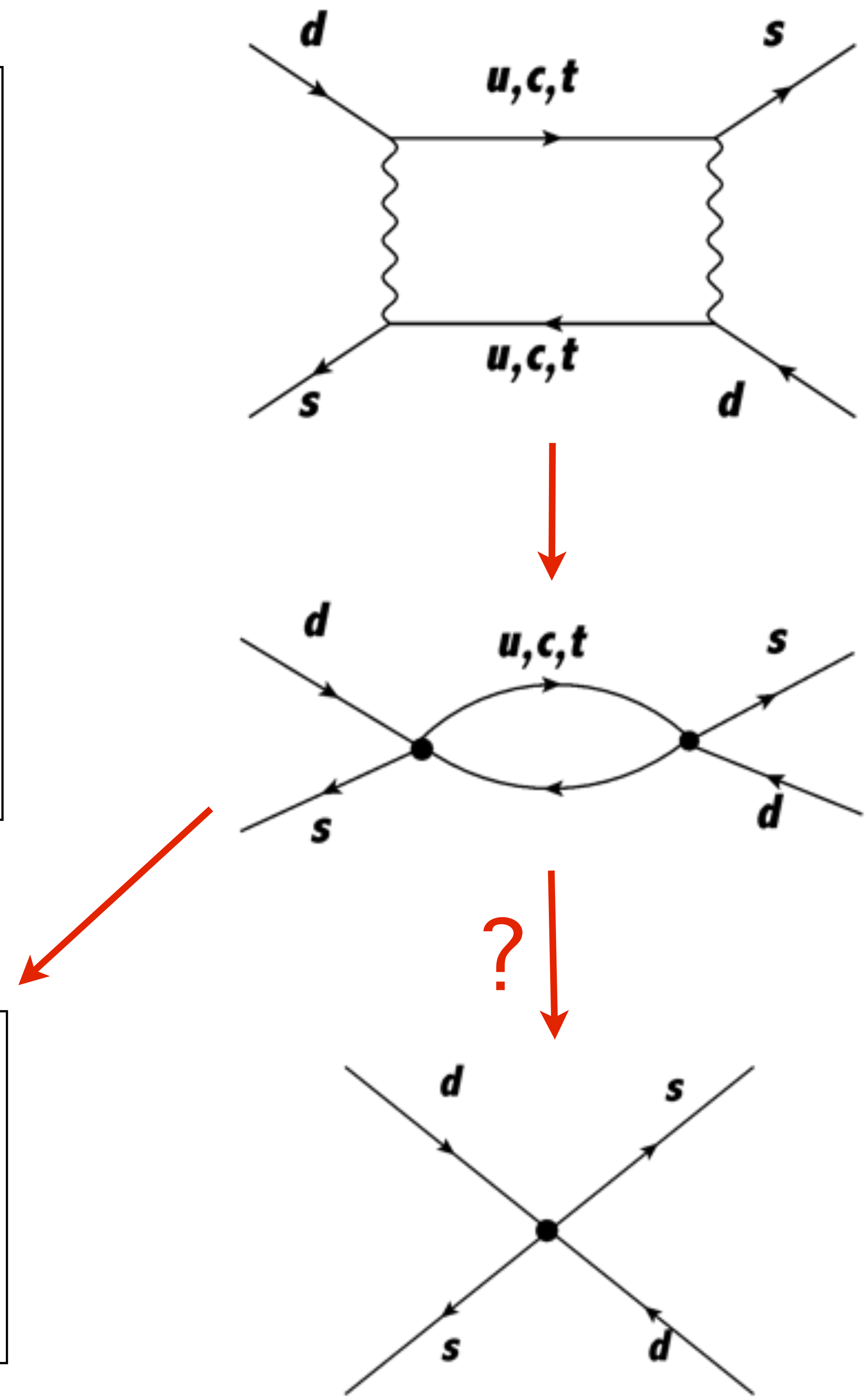
- Introduction
- Summary of the method
- Setup of the calculation
- Short distance effect
- Long distance effect
- Mass difference
- Conclusion and future plans

Introduction

- $K^0 - \bar{K}^0$ mixing :

  $\Delta M_K = 3.4583(6) \times 10^{-12}$ Mev

- Perturbative calculation can explain 70% of the mass difference
- Long distance effect



Directly evaluate second order weak process on a Lattice

Summary of the method

Neglect CP violation, K_L-K_S mass difference is given by :

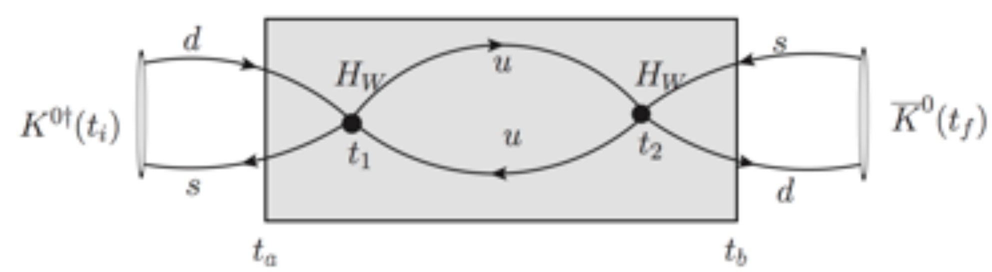
$$\Delta M_K = 2\mathscr{P}\int dE_n \frac{\langle \overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K - E_n}$$

Two parts to calculate ΔM_K :

✓ Evaluate lattice four point function

• Correct finite volume effect

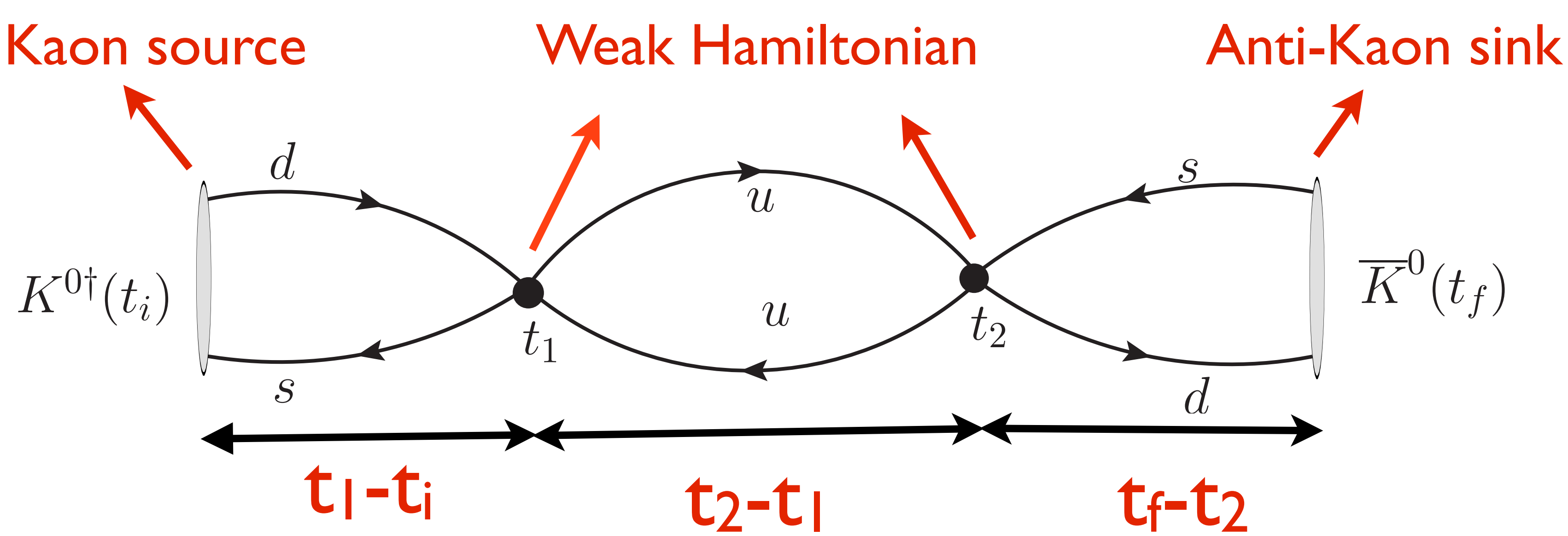
Principal part should be taken when dealing with $M_K = E_n$ singularity



Lattice four point function

Four point correlator :

$$G(t_f, t_1, t_2, t_i) = \langle \overline{K^0}(t_f) H_W(t_2) H_W(t_1) K^{0\dagger}(t_i) \rangle$$

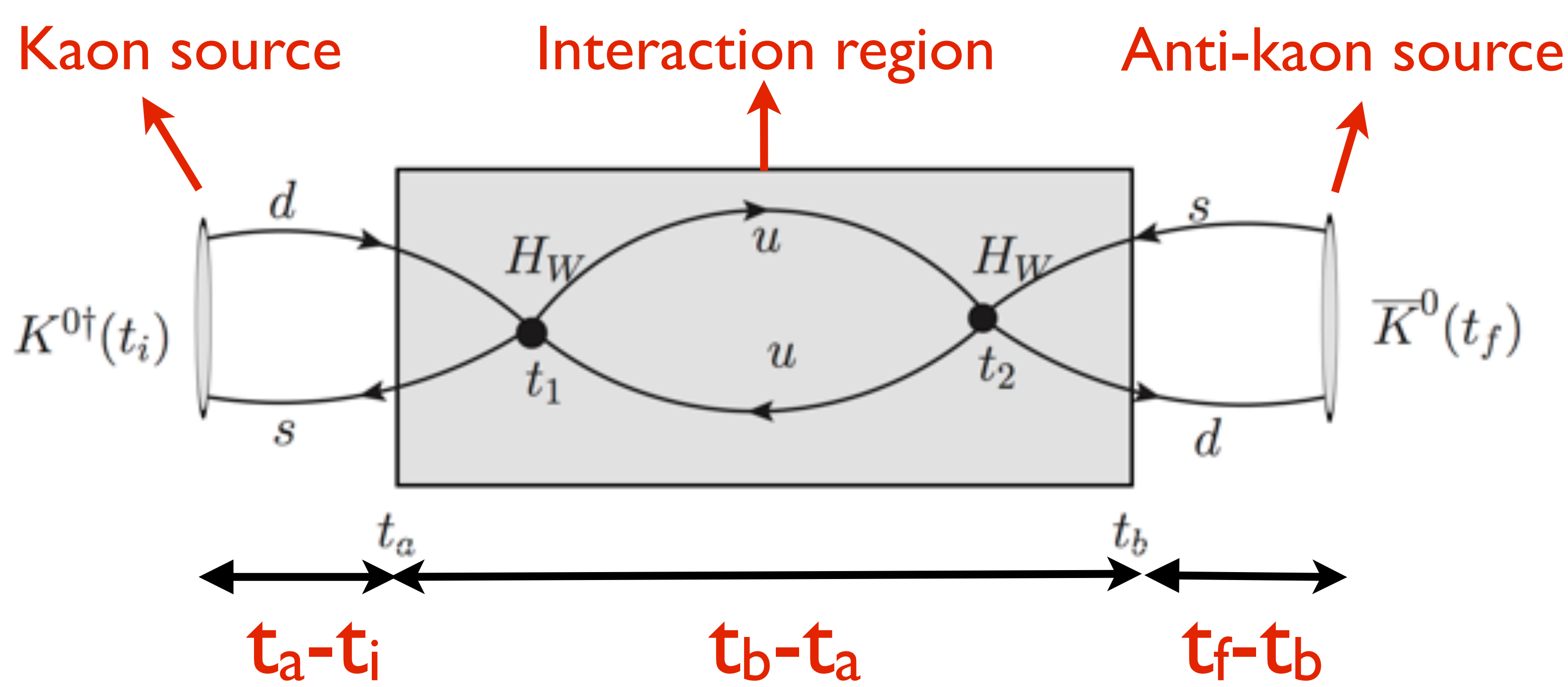


- t_1-t_i and t_f-t_2 should be sufficiently large to get a kaon
- Fix t_i and t_f, correlator depends only on t_2-t_1
- Refer to this quantity as unintegrated correlator

Integrated Correlator

Integrate the unintegrated correlator over a time interval :

$$\mathscr{A} = \frac{1}{2} \sum_{t_1=t_a}^{t_b} \sum_{t_2=t_a}^{t_b} \langle \overline{K^0}(t_f) H_W(t_2) H_W(t_1) K^{0\dagger}(t_i) \rangle$$



- t_f-t_b and t_a-t_i should be sufficiently large to get a kaon
- Fix t_i and t_f, correlator depends only on t_b-t_a
- Refer to this quantity as integrated correlator

After inserting a sum over intermediate states one obtains :

$$\mathscr{A} = N_K^2 e^{-M_K(t_f-t_i)} \left\{ \sum_{n \neq n_0} \frac{\langle \overline{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{M_K - E_n} \left(\underbrace{-T}_{1} - \underbrace{\frac{1}{M_K - E_n}}_{2} + \underbrace{\frac{e^{(M_K-E_n)T}}{M_K - E_n}}_{3,4} \right) \right.$$
$$\left. + \underbrace{\frac{1}{2} \langle \overline{K}^0 | H_W | n_0 \rangle \langle n_0 | H_W | K^0 \rangle T^2}_{5} \right\}$$

T = t_b-t_a+1 is the integration range, the terms in correlator fall into five categories:

1.Linear term, the coefficient gives finite volume approximation to ΔM_K

2. Constant term, which is trival

3. Exponential decreasing term, come from states E_n>M_K

4. Exponential increasing term, come from states E_n<M_K

5. Quadratic term, come from state E_n=M_K

Subtract from correlator

Correct finite volume effect

Finite Volume :

- ΔM_K is given by finite volume sum
- Tune lattice so $E_{\pi\pi} = M_K$
- Use degenerate perturbation theory, relate $E_{\pi\pi}$ with ΔM_K

Infinite volume :

- ΔM_K is given by infinite volume integral
- π-π phase shift relate to ΔM_K through kaon pole

Luscher condition:

$$\phi(E) + \delta_0(E) + \delta_W(E) = n\pi$$

Result for ΔM_K

Leading order term
- Tune volume so $M_K = E_{\pi\pi}$
- Remove $\pi\pi$ state

Add back finite volume correction

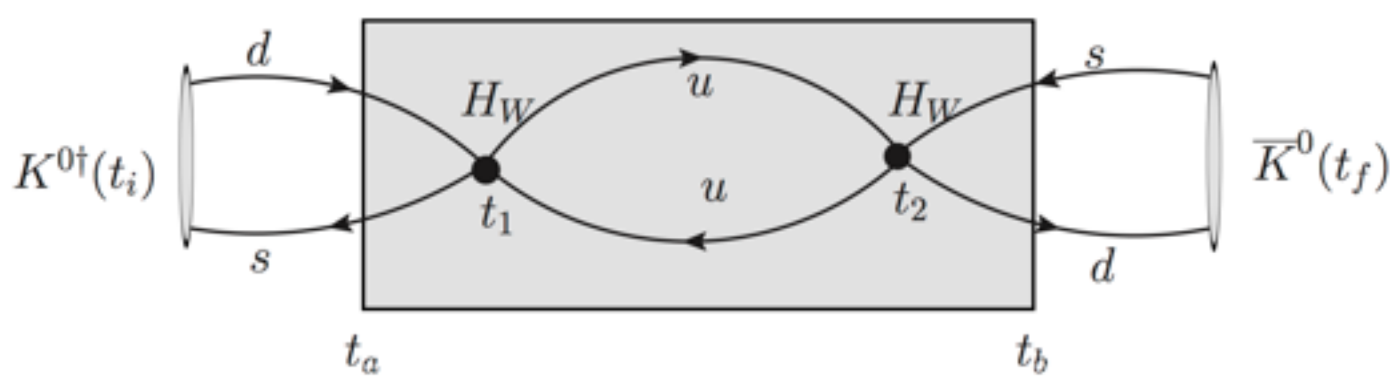
$$\Delta M_K = 2\sum_{n\neq n_0}\frac{\langle\overline{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{M_K - E_n} - \frac{\partial^2(\phi+\delta_0)/\partial E^2}{2\partial(\phi+\delta_0)/\partial E}|\langle n_0|H_W|K_S\rangle|^2 - \frac{\partial}{\partial E}|\langle n_0|H_W|K_S\rangle|^2$$

The finite volume correction is not done in this calculation

Setup of the calculation

Lattice ensemble :

- $16^3 \times 32 \times 16$, 2+1 flavor DWF
- Inverse lattice spacing 1.73(3) Gev
- M_π = 421 Mev, M_K = 559Mev
- 800 configurations, each separated by 10 time units



- Kaon wall sources at t_i = 0 at t_f = 27
- Weak Hamiltonian act between t_a=4 and t_b=23

Effective weak Hamiltonian

The ΔS=1 effective weak Hamiltonian in a 4 flavor theory :

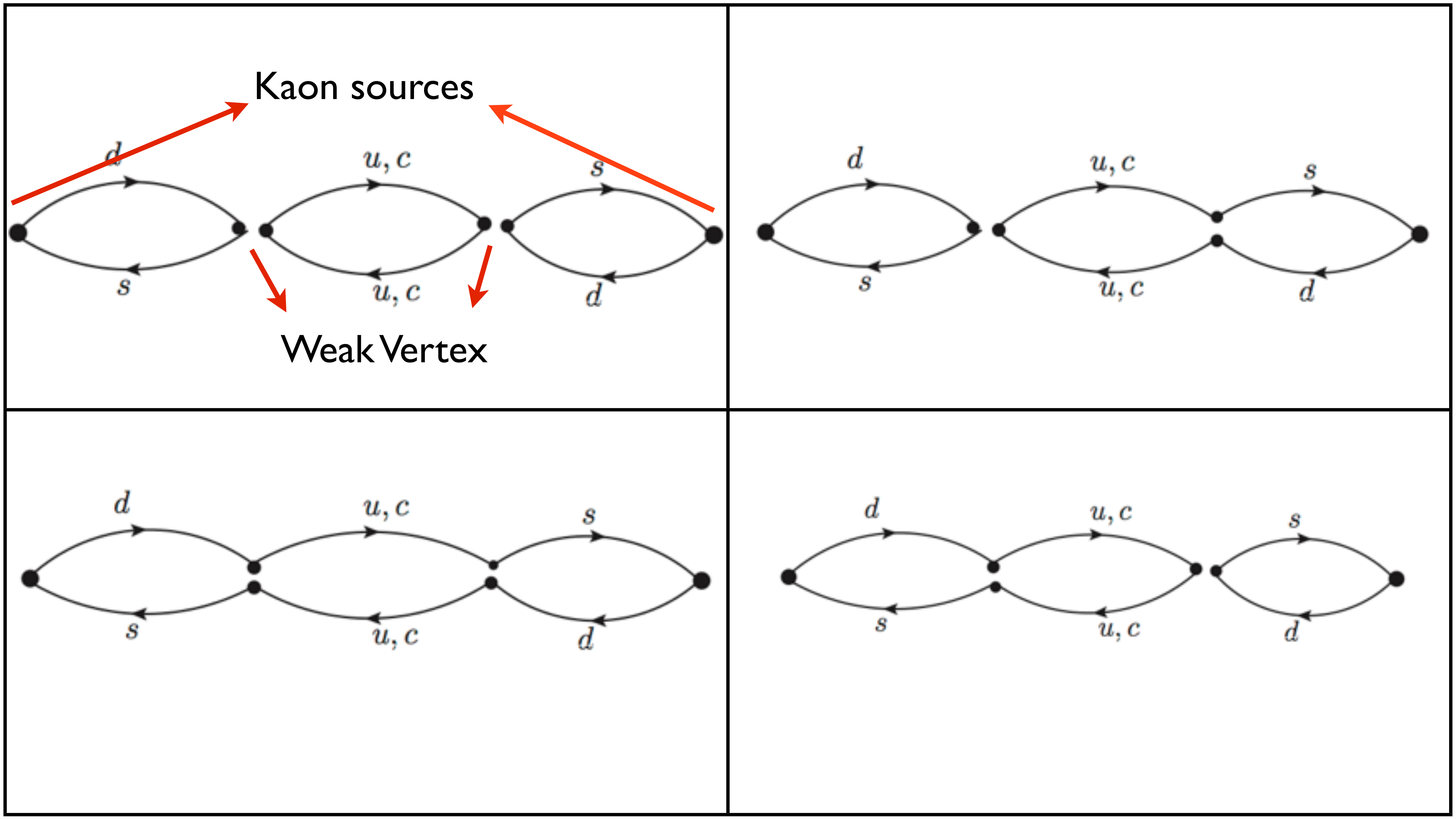
$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'})$$

Here we only include current-current operators :

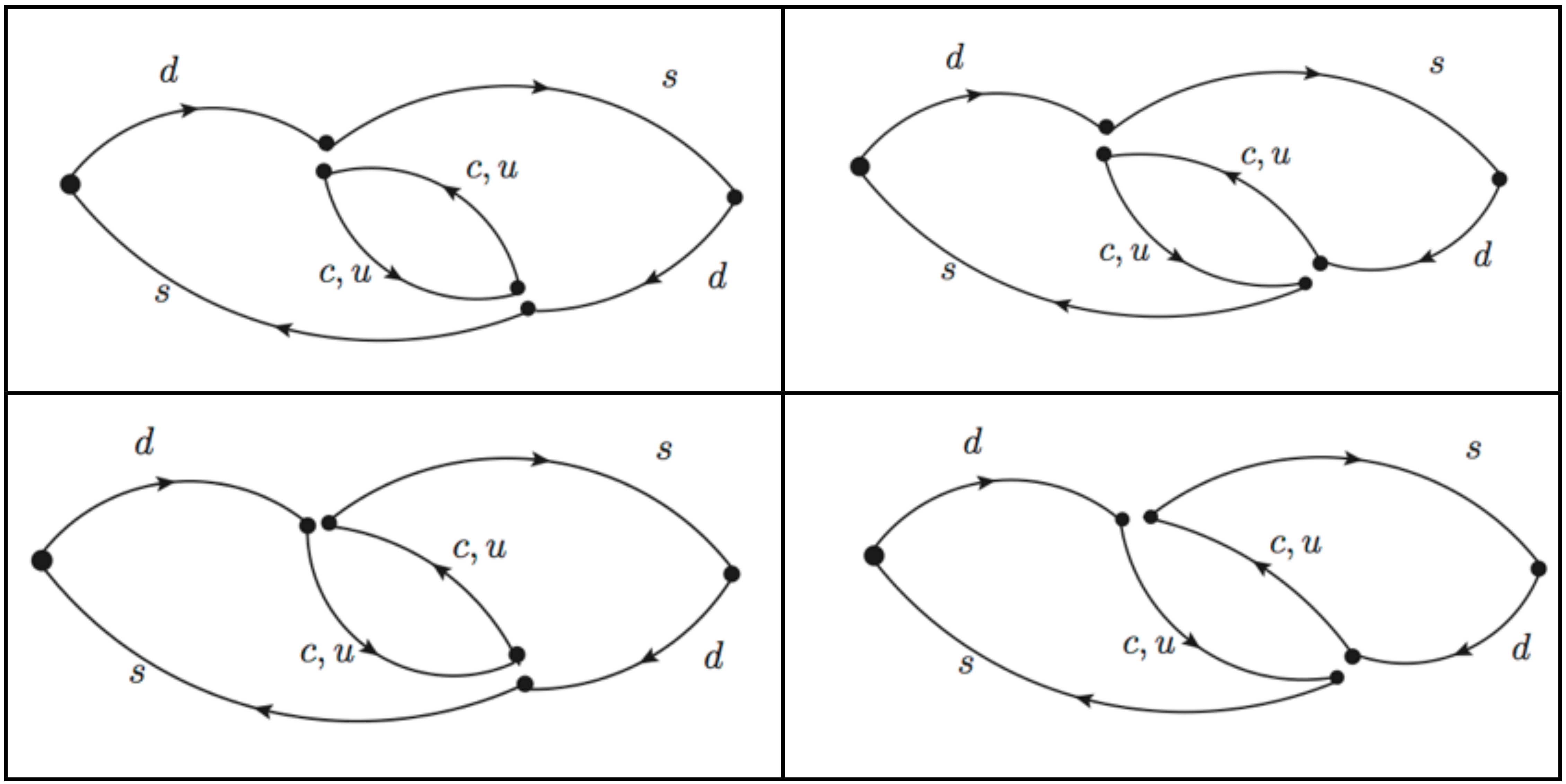
$$Q_1^{qq\prime} = (\bar{s}_i d_i)_{V-A} (\bar{q}_j q'_j)_{V-A}$$
$$Q_2^{qq\prime} = (\bar{s}_i d_j)_{V-A} (\bar{q}_j q'_i)_{V-A}$$

All the penguin operators are neglected, since they are highly suppressed because of GIM cancellation

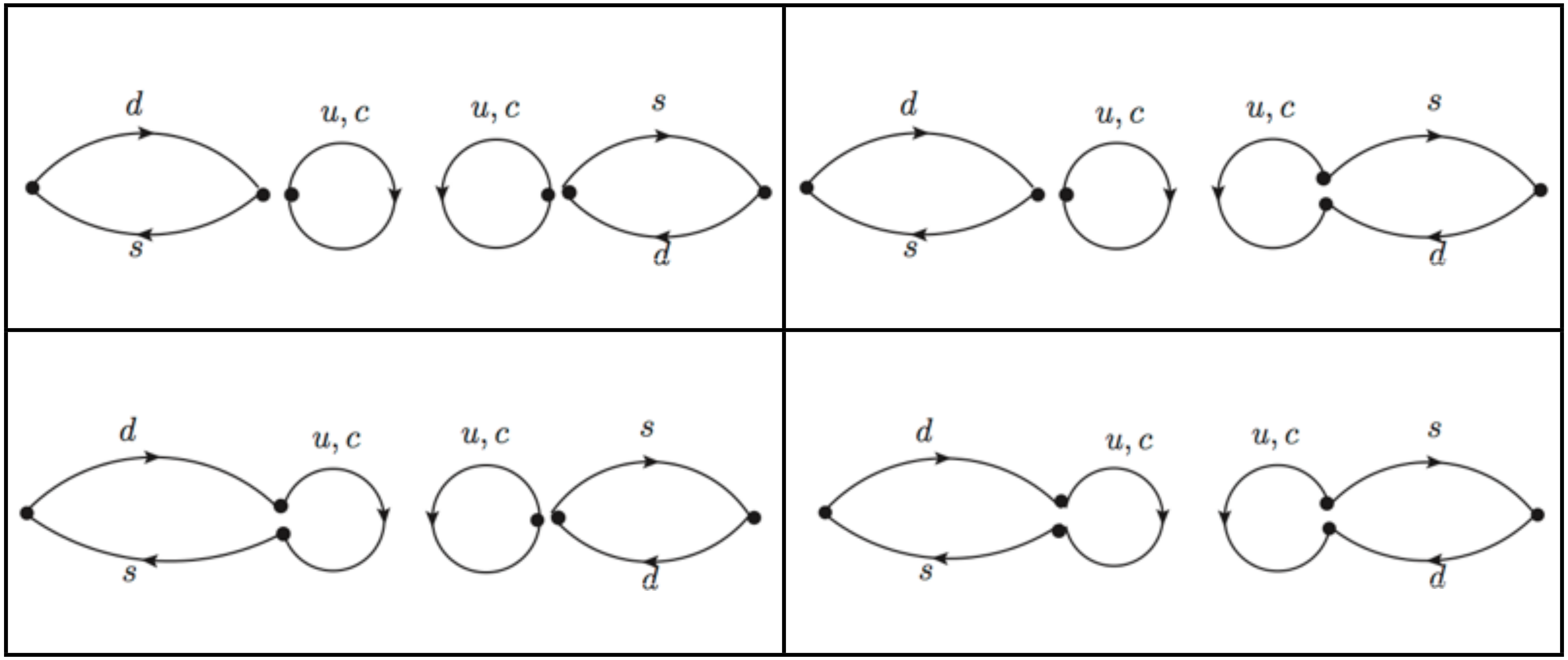
Type 1 diagrams



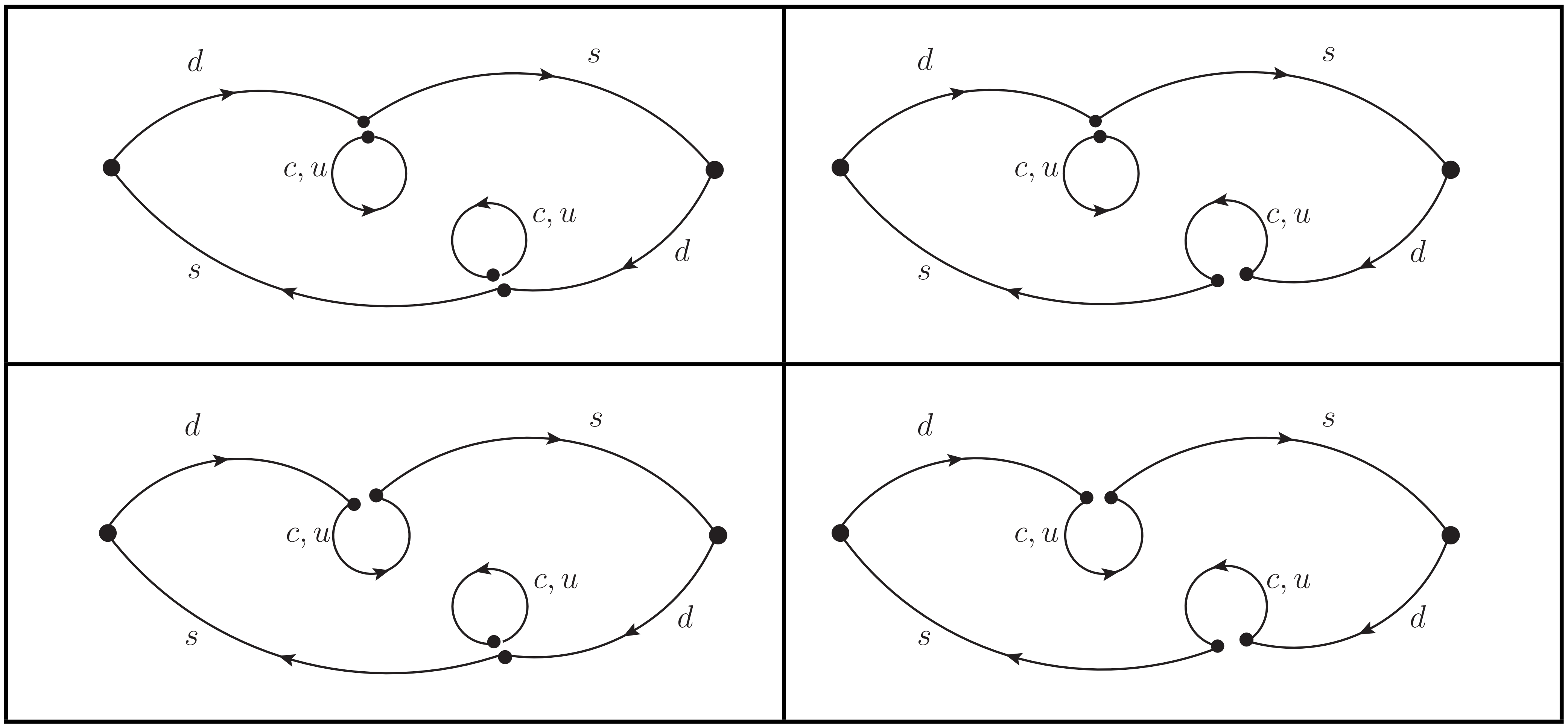
Type 2 diagrams



Type 3 digrams, not calculated

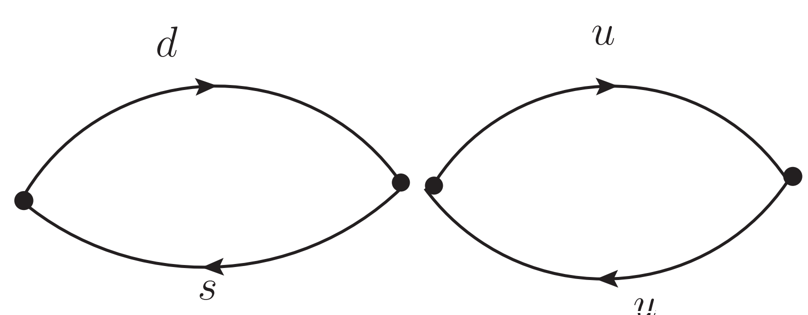


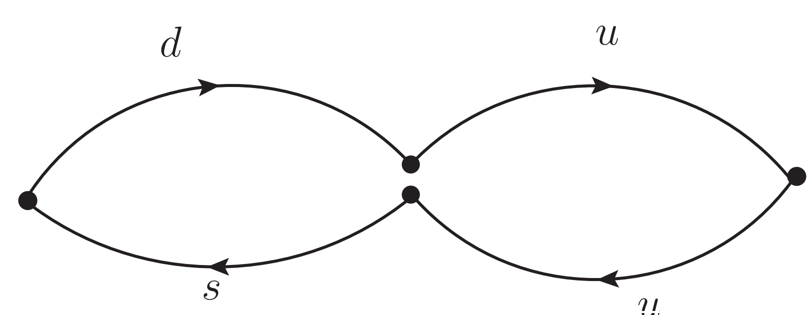
Type 4 diagrams, not calculated



π^0 intermediate state

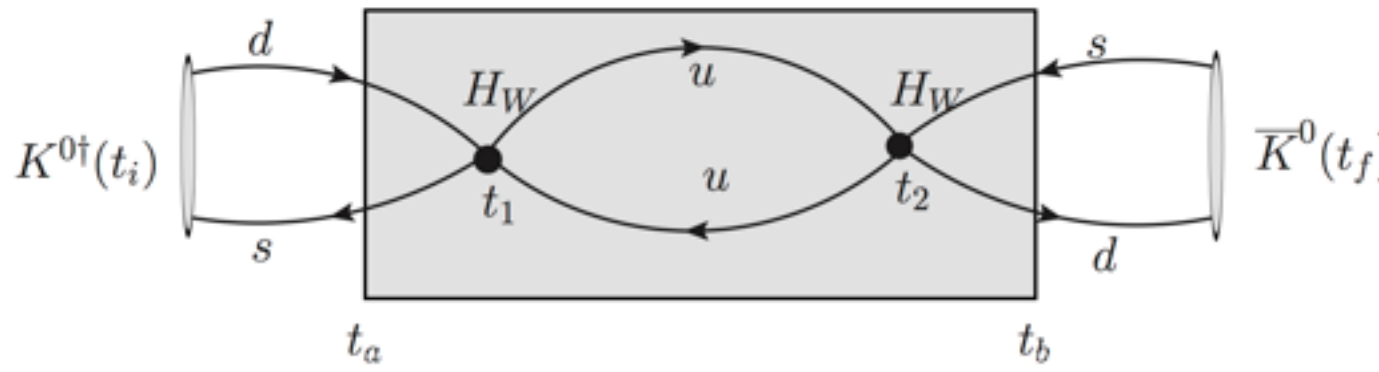
π^0 intermediate state contributes an exponentially increasing term in the integrated correlator, which must be identified and removed. In this non unitary calculation, π^0 and η have same mass, since only up quark can appear in our intermediate state,we define $\pi^0 = i\bar{u}\gamma_5 u$, calculate following diagrams to compute kaon to pion decay amplitude :

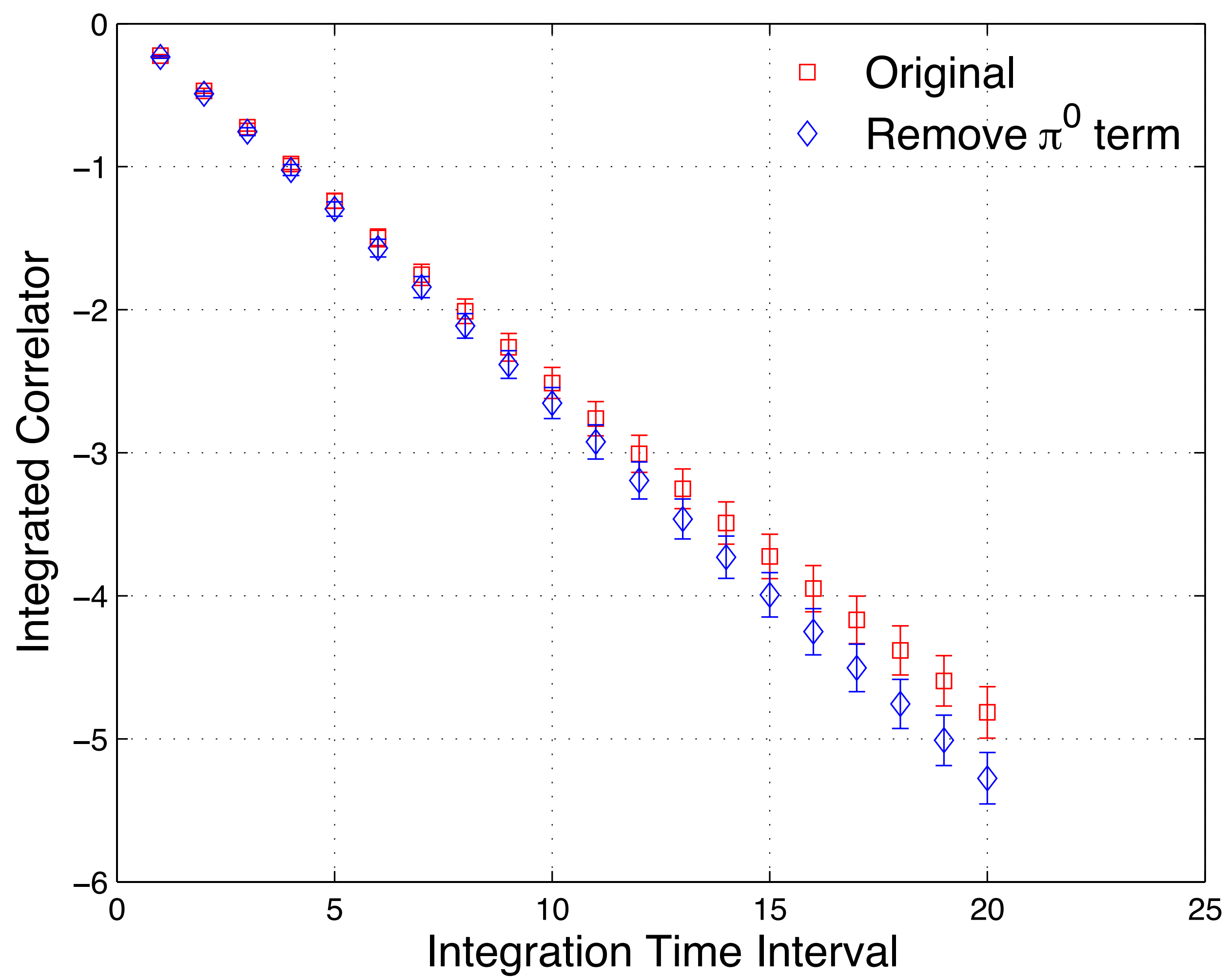




Result without GIM cancellation

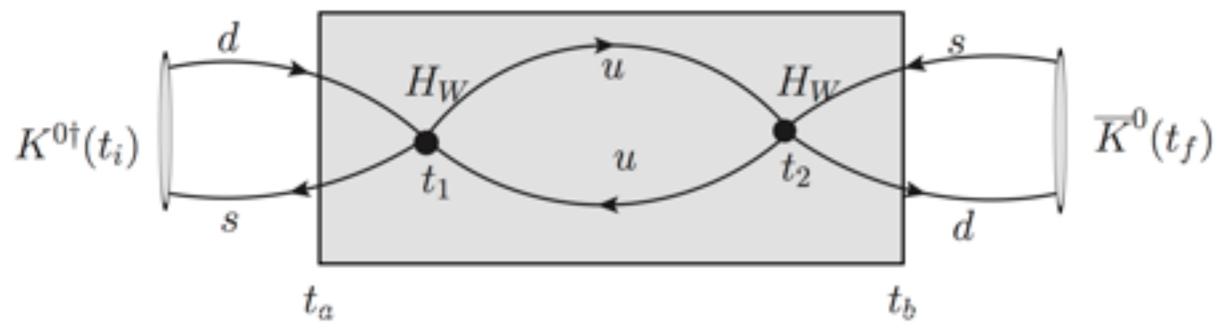
- Both operators are Q_1
- Without GIM, there will be divergent short distance effect
- The dependence of correlator on time is almost linear imply that largest contribution comes from short distance



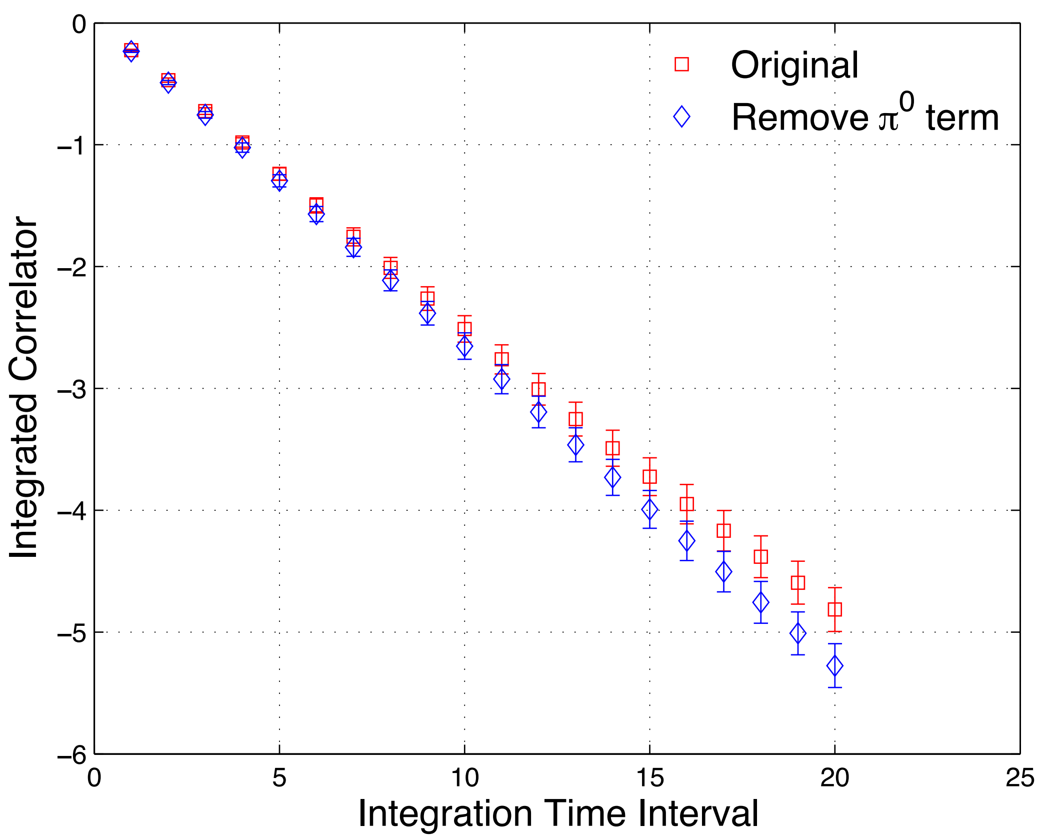


Artificial cutoff

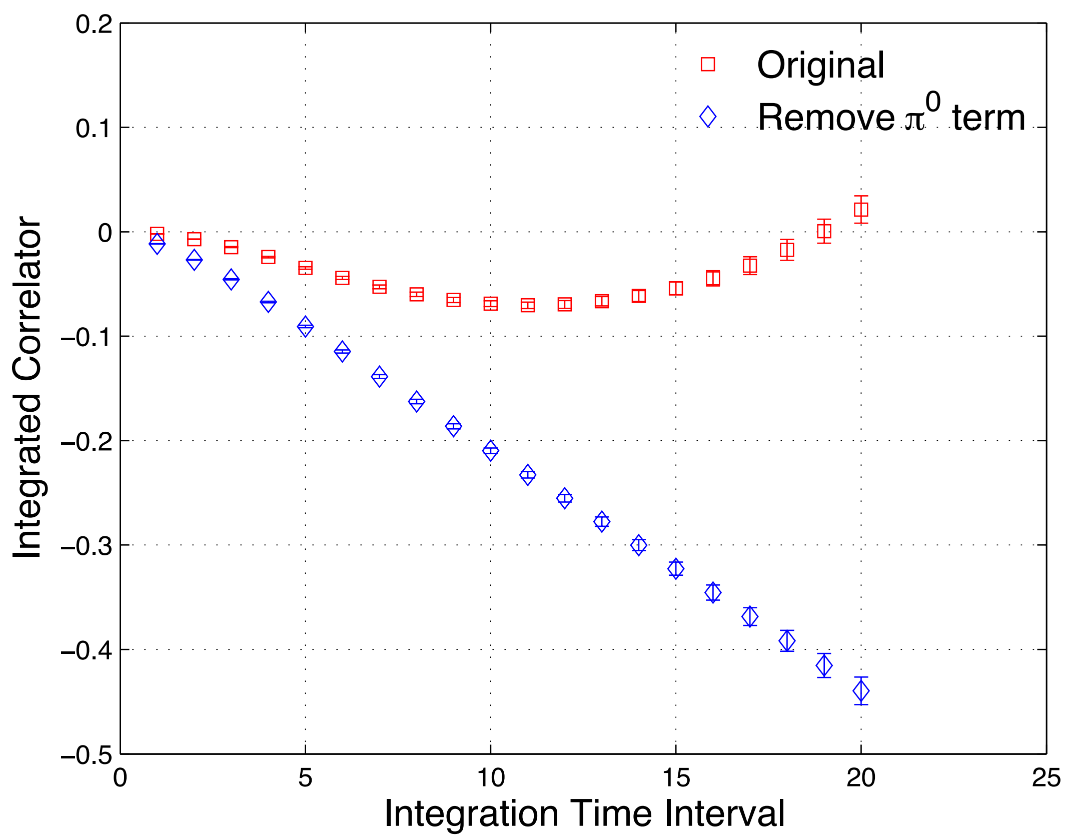
We can impose an artificial cutoff, require $|x_2 - x_1| \geq r$ while doing integral :



No cutoff



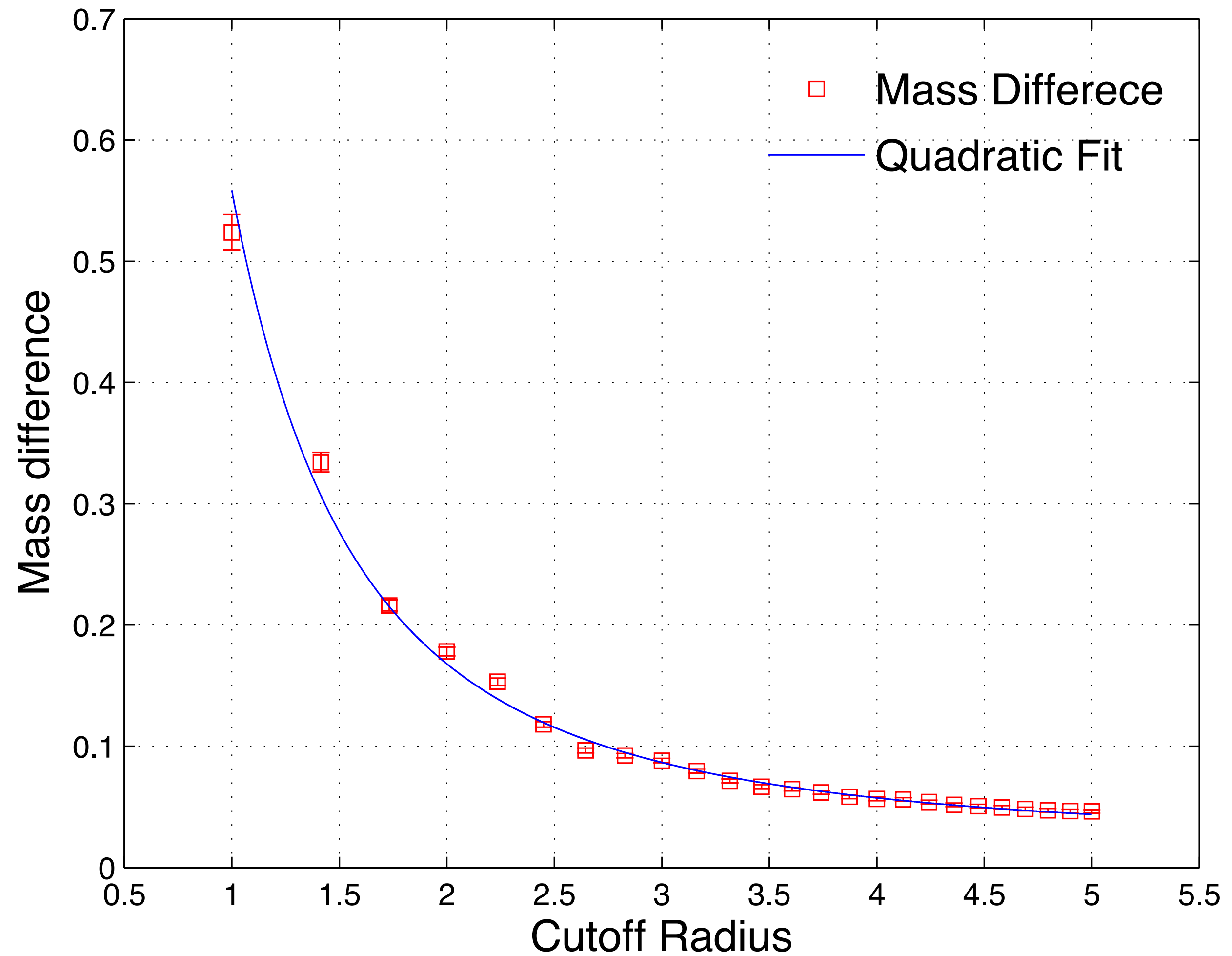
Cutoff = 5



Quadratic divergence

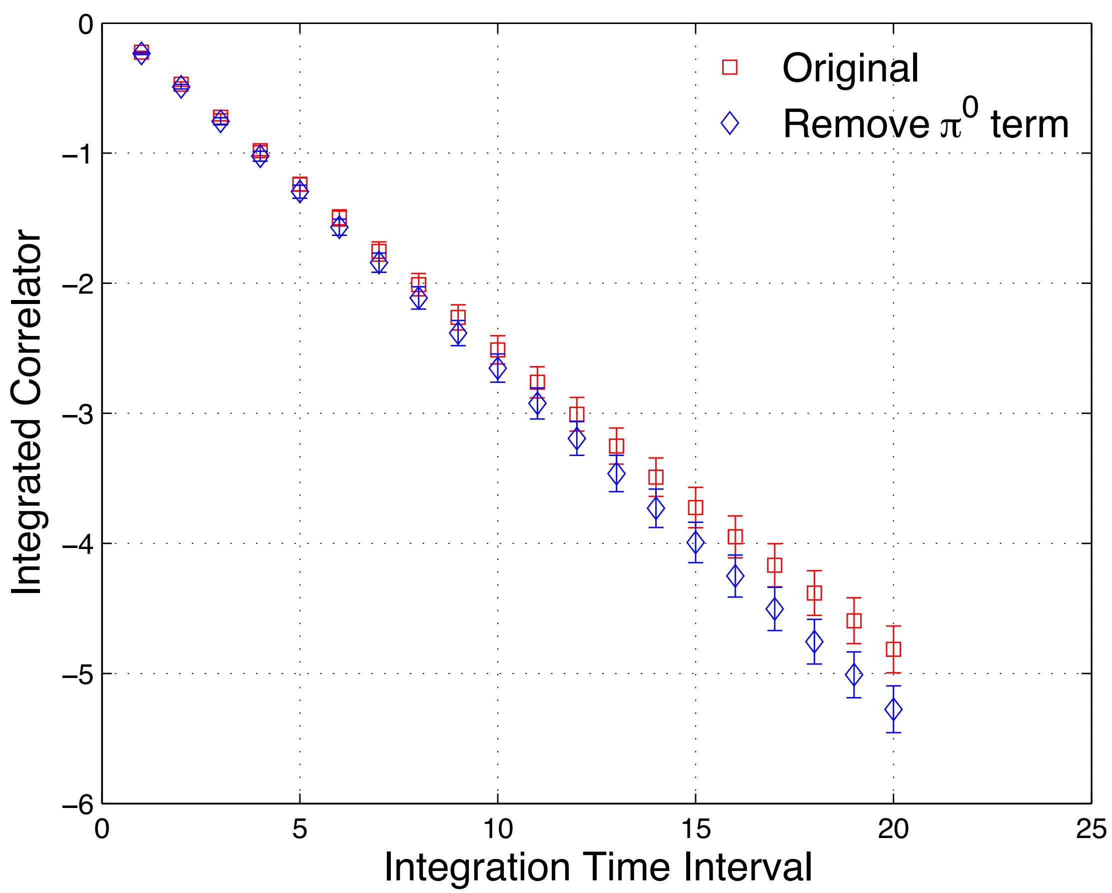
Mass difference is given by the slope of integrated correlator plot while the integration range is large enough.

$$\Delta M_K(R) = \frac{a}{R^2} + b$$

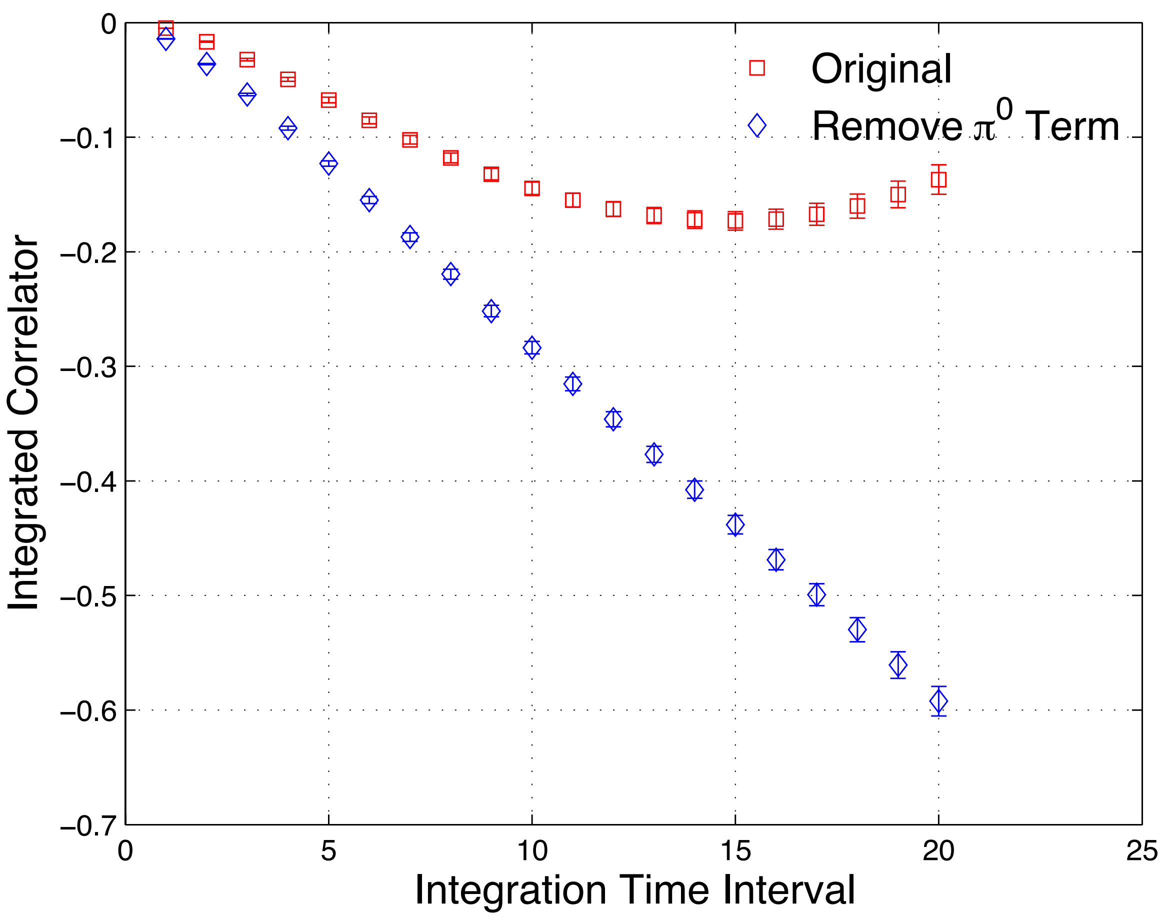


GIM remove the divergence in short distance :

No charm quark

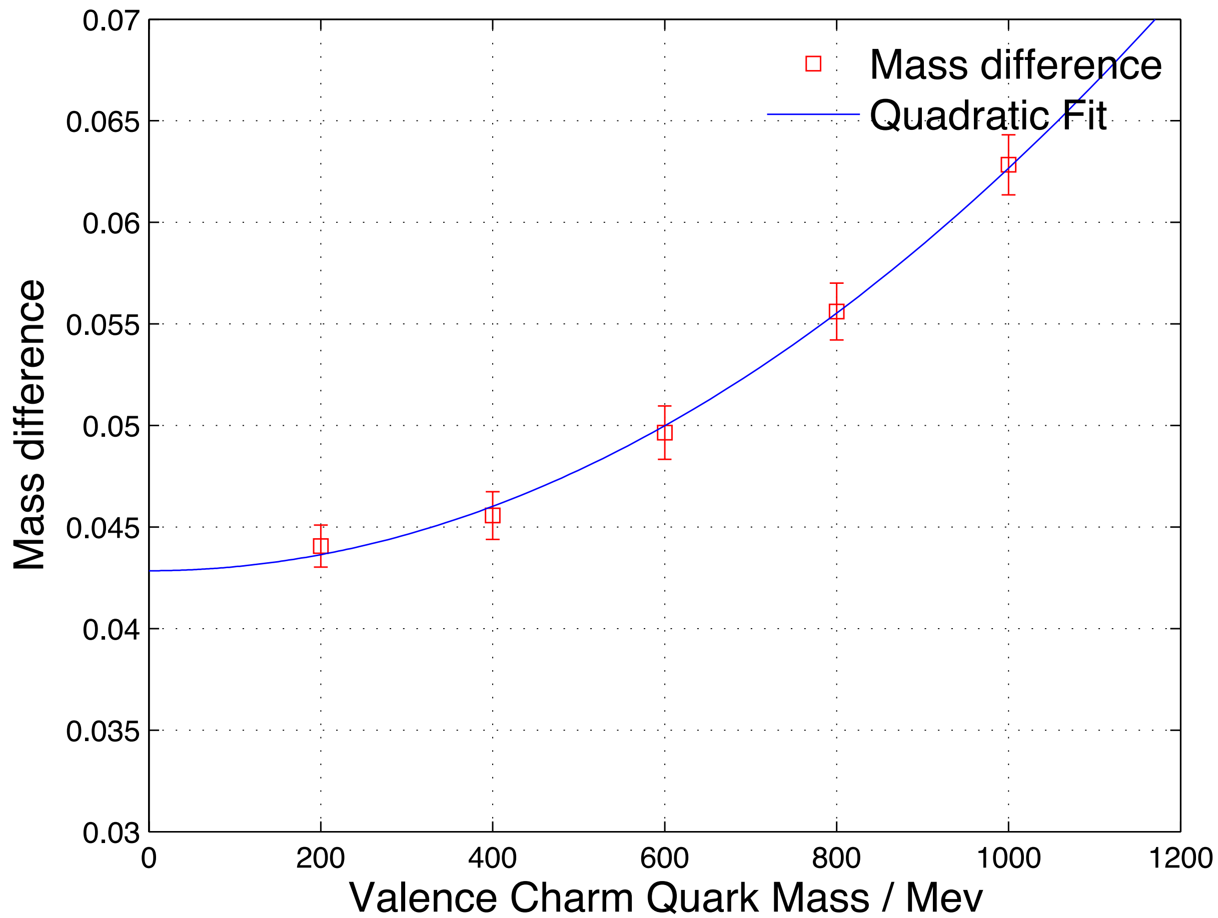


1 Gev charm quark



Quadratic Fit :

$$\Delta M_K(m_c) = a\, m_c^2 + b$$



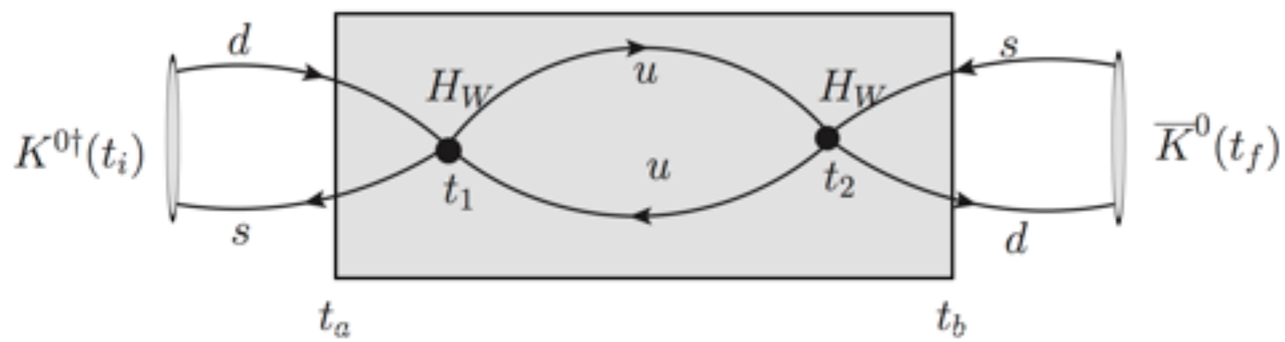
Is 1 Gev charm too heavy ?
Quadratic dependence on m_c will be cutoff buy lattice spacing if charm is too heavy.
The fitting result suggest we haven't reach that region

Long distance effect

Investigate unintegrated correlator :

$$G(T; t_i, t_f) = N_K^2 e^{-M_K(t_f - t_i)} \sum_n \langle \overline{K^0} | H_W | n \rangle \langle n | H_W | K^0 \rangle e^{-(E_n - M_K)T}$$

Here T is the time separation between to weak Hamiltonian

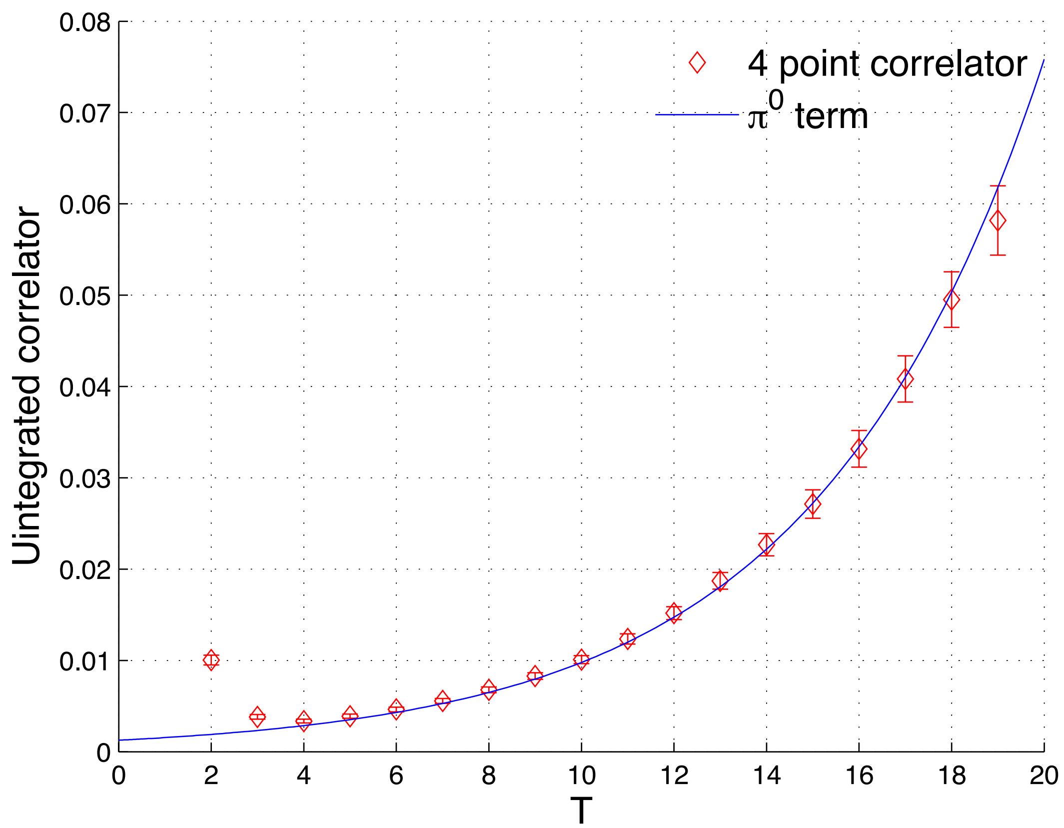


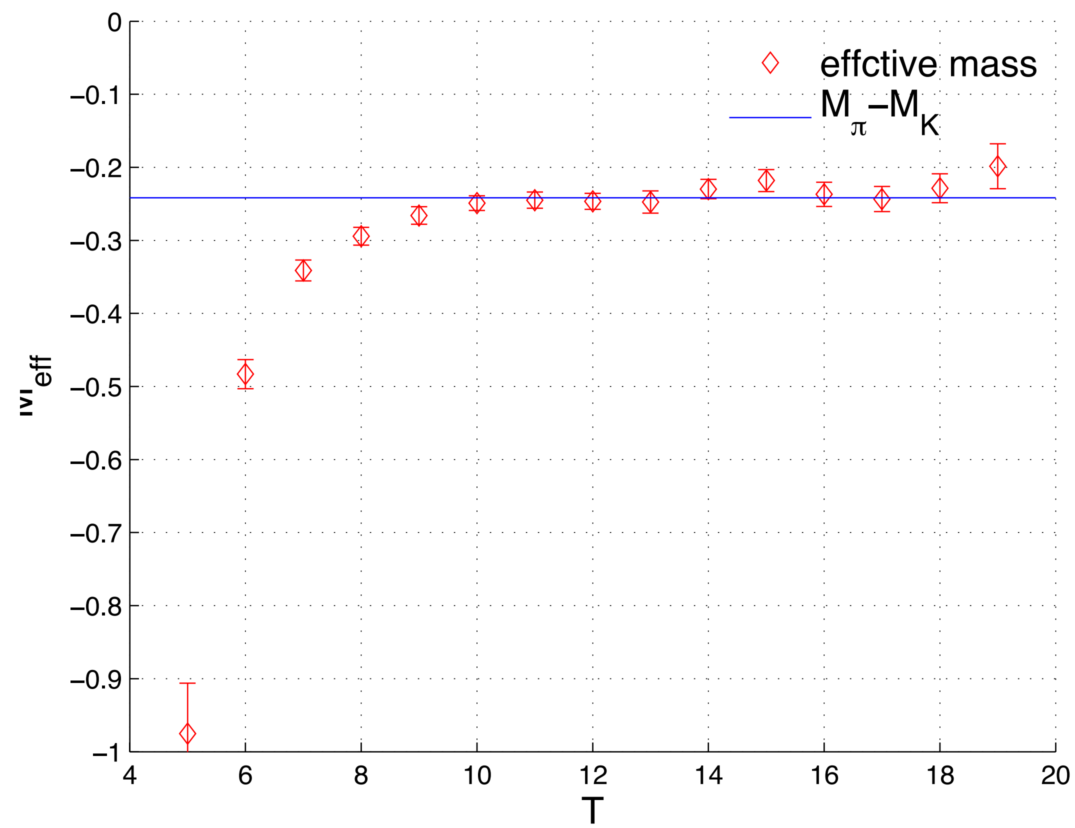
- Separate the Hamiltonian into two parity channel :
  - Parity conserving channel, long distance effect dominate by π^0 intermediate state
  - Parity violating channel, long distance effect dominate by $\pi\pi$ intermediate state
- Use various kaon masses

Parity conserving channel

In long distance, correlator dominate by π^0 term :

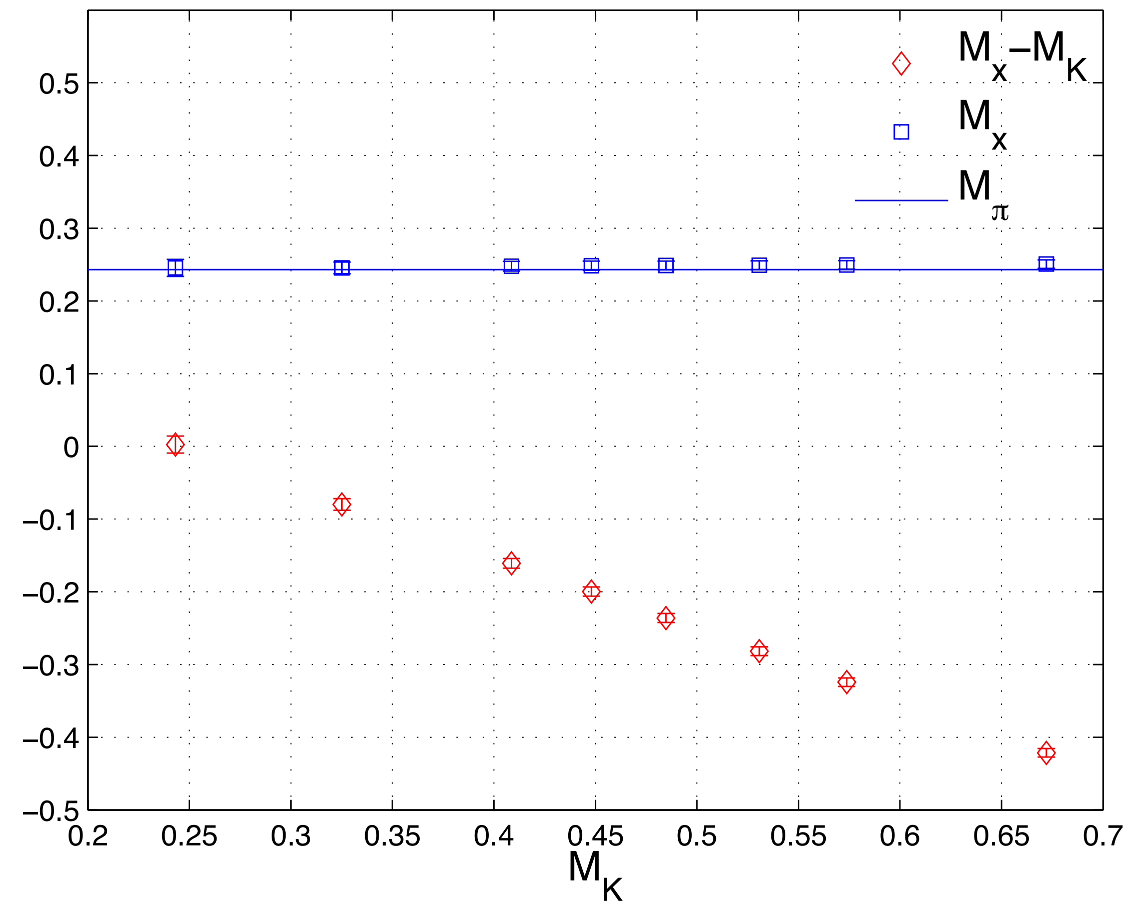
$$G(T;t_i,t_f) = N_K^2 e^{-M_K(t_f-t_i)} \langle \overline{K^0} | H_W | \pi^0 \rangle \langle \pi^0 | H_W | K^0 \rangle e^{-(E_\pi - M_K)T}$$





Parity conserving channel

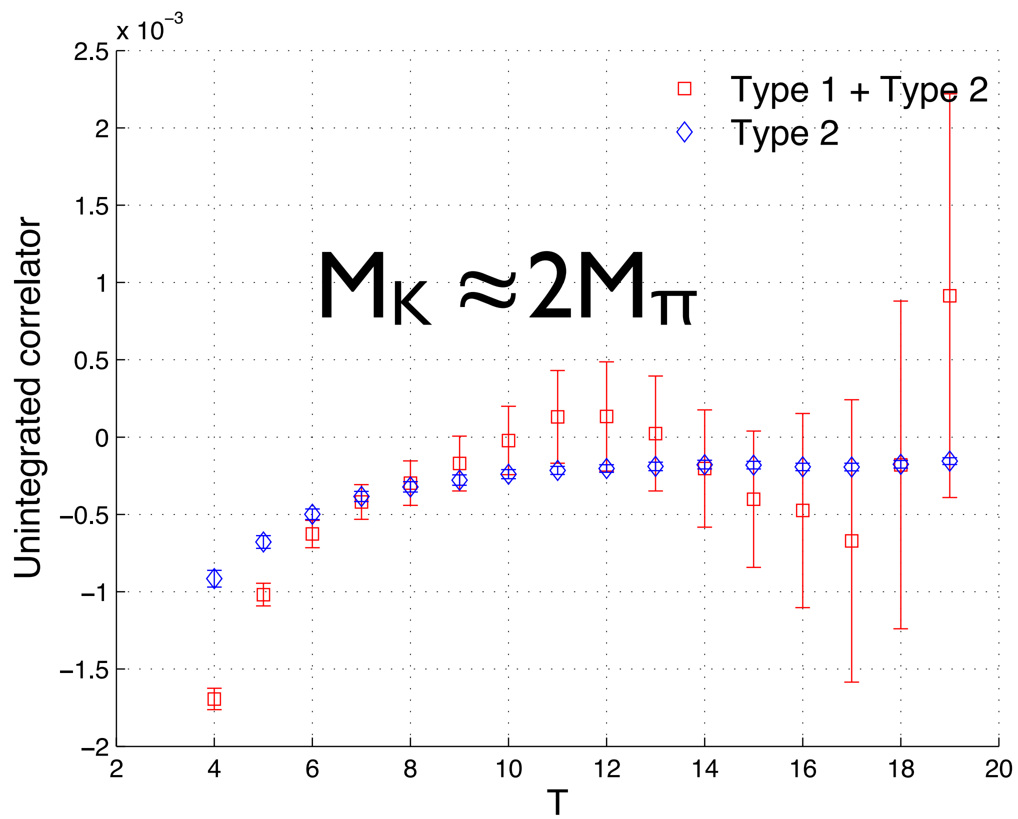
- Points are the fitting results from unintegrated correlators at various kaon masses
- Horizontal line is the "exact" pion mass given by two point correlator calculation



Parity violating channel

In long distance, correlator dominate by ππ term :

$$G(T;t_i,t_f) = N_K^2 e^{-M_K(t_f-t_i)} \langle \overline{K^0}|H_W|\pi\pi\rangle\langle\pi\pi|H_W|K^0\rangle e^{-(E_{\pi\pi}-M_K)T}$$

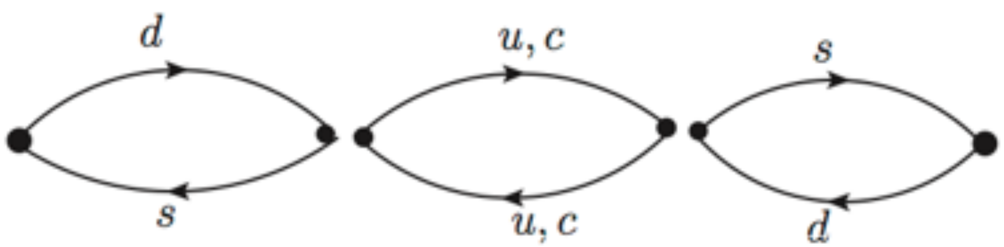


We expect to see plateau at long distance :

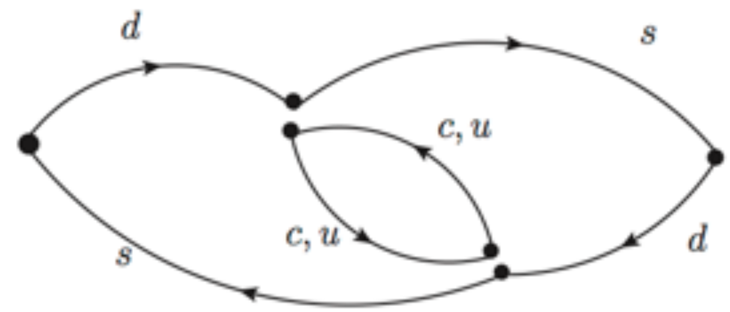
- No signal at long distance
- Good signal from type 2 diagrams only

Parity violating channel

Type 1



Type 2

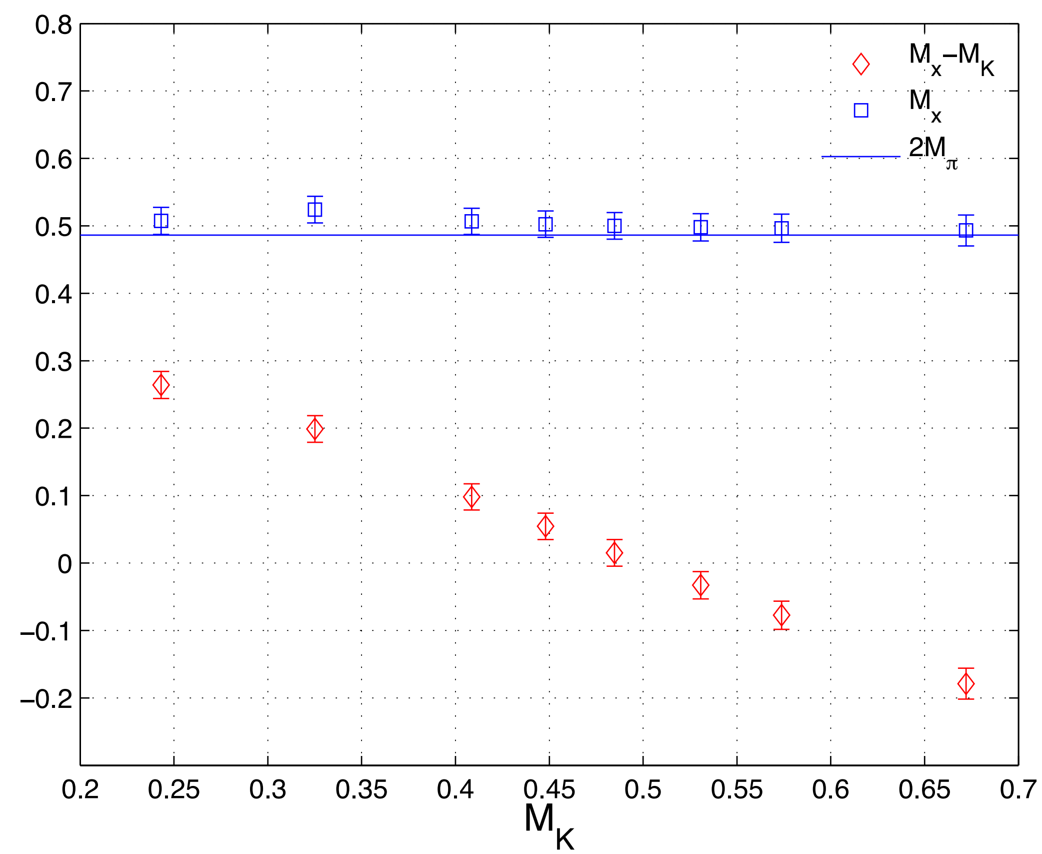


Noise behave like π ,exponentially increasing noise to signal ratio

The signal from type 2 contractions don't have such noise

Parity violating channel

- Points are the fitting results from unintegrated correlators at various kaon masses, type 2 contractions only
- Horizontal line is the "exact" 2 pion mass given by two point correlator calculation



Mass difference

M_π = 421 Mev
M_c = 1 Gev

- Only included statistical error
- Finite volume effect not corrected

$$\boxed{\Delta M_K^{exp} = 3.483(6) \times 10^{-12} \text{ Mev}}$$

M_K (Mev)	ΔM_K^{11}	ΔM_K^{12}	ΔM_K^{22}	ΔM_K ($\times 10^{-12}$ Mev)
563	6.38(14)	-2.64(14)	1.47(8)	5.52(24)
707	8.90(21)	-2.96(23)	2.10(12)	7.38(37)
775	10.63(27)	-3.18(30)	2.48(15)	8.61(49)
839	12.56(34)	-3.62(40)	2.89(20)	9.93(65)

$\approx 2M_\pi$

Conclusions and future plans

- Lattice calculation of ΔM_K is possible :
  - ✓ Use GIM to remove divergence in short distance
  - ✓ Remove π exponentially term
  - Use on-shell $K \rightarrow \pi\pi$ kinematics, remove quadratic term from integrated correlator
  - Add finite volume correction term
- Include type 3 and type 4 diagrams in future
- Use Low mode averaging or A2A to collect statistics more efficiently

Operator mixing and renormalization

Three group of operators :

$$\tilde{Q}_1 = (\bar{s}_i d_i)_{V-A}(\bar{u}_j u_j)_{V-A} - (\bar{s}_i d_i)_{V-A}(\bar{c}_j c_j)_{V-A}$$

$$\tilde{Q}_2 = (\bar{s}_i d_j)_{V-A}(\bar{u}_j u_i)_{V-A} - (\bar{s}_i d_j)_{V-A}(\bar{c}_j c_i)_{V-A}$$

$$Q_1^{cu} = (\bar{s}_i d_i)_{V-A}(\bar{c}_j u_j)_{V-A}$$

$$Q_2^{cu} = (\bar{s}_i d_j)_{V-A}(\bar{c}_j u_i)_{V-A}$$

$$Q_1^{uc} = (\bar{s}_i d_i)_{V-A}(\bar{u}_j c_j)_{V-A}$$

$$Q_2^{uc} = (\bar{s}_i d_j)_{V-A}(\bar{u}_j c_i)_{V-A}$$

Equivalent basis :

$$Q_+^X = Q_1^X + Q_2^X \qquad (84,1)$$

$$Q_-^X = Q_1^X - Q_2^X \qquad (27,1)$$

X= ~, cu, uc   SU(4)×SU(4)

- Operators will not mix with penguin
- Renormalization for three groups of operators should be identical

Correct finite volume effect

- Singular energy denominator $1/(M_K - E_n)$ will introduce uncontrolled errors
- Use generalized Lellouch-Lucsher method :
  - Tune lattice so $E_{\pi\pi} = M_K$
  - Finite volume $E_{\pi\pi}$ depend on finite volume sum
  - Infinite volume π-π resonant phase shift δ_W depend on infinite volume integral
  - Luscher condition relate them :

$$\phi(E) + \delta_0(E) + \delta_W(E) = n\pi$$

Finite volume energy

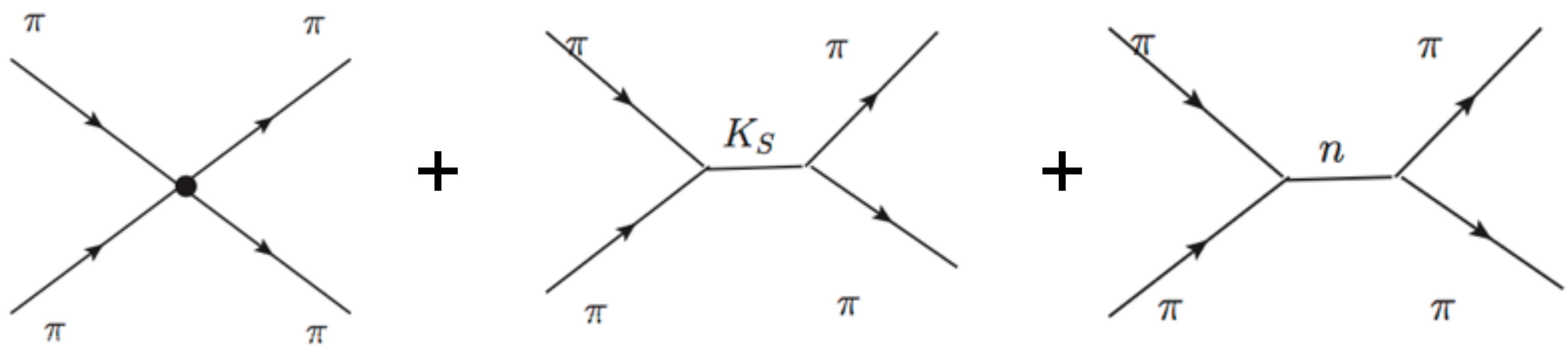
- Let $|n_0\rangle$ be the π-π state degenerate with kaon
- Second order perturbation theory

$$\begin{pmatrix} M_K + \sum_{n\neq n_0} \frac{|\langle n|H_W|K\rangle|^2}{M_K - E_n} & \langle K|H_W|n_0\rangle \\ \langle n_0|H_W|K\rangle & E_{n_0} + \sum_{n\neq K} \frac{|\langle n|H_W|n_0\rangle|^2}{E_{n_0} - E_n} \end{pmatrix}$$

- π-π state energy is given by :

$$E_\pm = M_K \pm \langle K|H_W|n_0\rangle + \frac{1}{2}\left\{ \sum_{n\neq n_0} \frac{|\langle n|H_W|K\rangle|^2}{M_K - E_n} + \sum_{n\neq K} \frac{|\langle n|H_W|n_0\rangle|^2}{E_{n_0} - E_n} \right\}$$

Infinite volume scattering



Strong interaction Kaon pole Weak interaction

Total phase shift is given by :

$$\delta(E) = \delta_0(E) + arctan(\frac{\Gamma(E)/2}{M_K + \Delta M_K - E}) - \pi \sum_{n \neq K} \frac{\langle n|H_W|\pi\pi\rangle|^2}{E - E_n}$$

Require that :

$$\delta(E_{\pm}) + \phi(E_{\pm}) = n\pi$$